Lab: AP Review Sheets

Chapter 11: Rotational Motion

AP Physics, Period G
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Background / Summary:
This sheet reviews rotational motion, moment of inertia, parallel axis theorem, and torque.

Useful Equations and Relationships

Linear and Rotational Relationships:	Rotational Big 4:
• $\Delta s = \Delta\theta r$ • $v_{tan} = \omega r$ • $a_{tan} = \alpha r$ ** these relationships work even if acceleration is not constant	• $\omega = \omega_o + \alpha t$ • $\Delta\theta = \frac{1}{2}(\omega_o + \omega)t$ • $\Delta\theta = \omega_o t + \frac{1}{2}\alpha t^2$ • $\omega_2 = \omega_{0,2} + 2\alpha\Delta\theta$
Rotational form of Newton's Second Law:	$\sum F = ma \Rightarrow \sum \tau = I\alpha$
Rotational KE Where ***I*** is the moment of inertia	$K_{rotational} = \frac{1}{2}I\omega^2$
Moment of Inertia ***I*** is analogous to the linear quantity ***m***, and is basically an objects resistance to change motion. r is the distance from the center of mass to the axis of rotation.	$I = \sum m_i r_i^2$

Use these relationships to calculate the moment of inertia for different geometries:

$$I = \int r^2 dm \qquad \begin{aligned} dm &= \lambda \ dr \\ dm &= \sigma \ dA \\ dm &= \rho \ dV \end{aligned}$$

You can look online for the derivations, but here are the moment of inertias for three common geometries. Each is assumed to be oriented <u>about the central axis.</u> If its not, just change the limits of your integral accordingly or use the parallel axis theorem.

Uniform **Hoop** of mass M and radius R	$I = MR^2$
Uniform **rod** of mass M with length L at its center of mass	$I = \frac{1}{12}ML^2$
Uniform **solid cylinder** with mass M, radius R, and length L	$I = \frac{1}{2}MR^2$

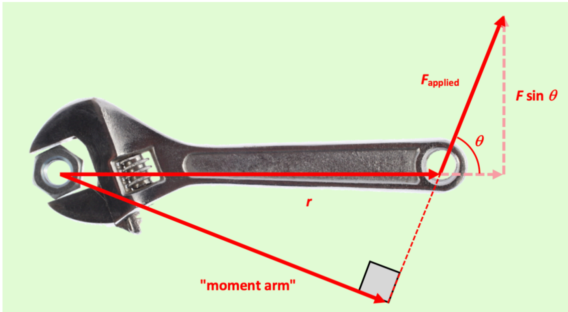
Parallel Axis Theorem: $I = I_{cm} + MD^2$

If one knows the moment of inertia about the center of mass of an object, one can determine the moment of inertia about any other axis parallel to the original one.

Torque:

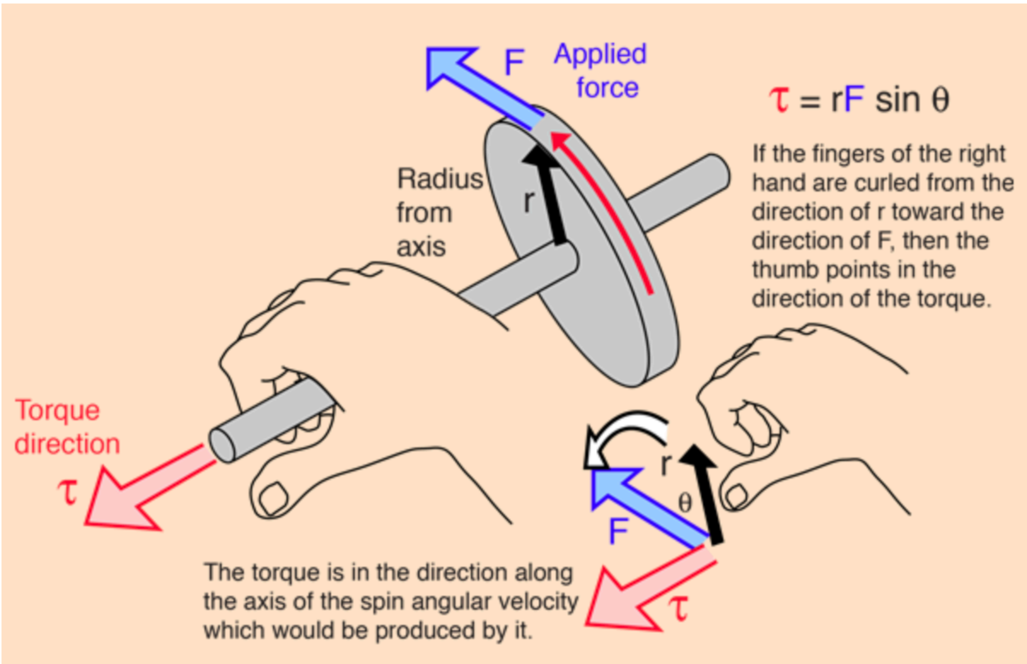
Torque is the rotational equivalent of force. Force is the ability to cause an acceleration of an object. Torque is the ability of a force to cause an angular acceleration of an object.

- Torque is a vector.
- r is the distance from the axis of rotation to the location on the object the force is applied (moment arm)
- F is the magnitude of the force.
- θ is the angle between r and F.



Three equations to solve torque:
$\lvert\tau\rvert = \lvert r\rvert \lvert F\rvert \sin\theta$
$\lvert\tau\rvert = \lvert F_{\perp}\rvert \lvert r\rvert$
$\lvert\tau\rvert = r_{\perp} \lvert F\rvert$

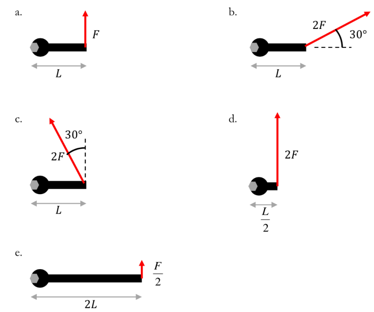
Right hand rule for Torque



Practice Problems:

1. **[Easy]** Find the cross product of 5i + 6j and -3i + -2j using determinants.

2. **[Medium]** A series of wrenches of different lengths is used on a hexagonal bolt, as shown below. Which combination of wrench length and Force applies the greatest torque to the bolt?



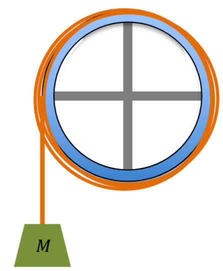
3. **[Hard]** A wheel is mounted to a frictionless fixed axle and suspended from a vertical support. Several turns of light cord are wrapped around the wheel, and a mass M is attached to the end of the cord and allowed to hang. The mass is released from rest

a) Determine the tension in the cord supporting the mass as it accelerates downwards.

b) Calculate the angular acceleration of the wheel as the mass descends

c) Determine the instantaneous velocity of the mass after the wheel has turned one revolution.

d) Determine the instantaneous angular momentum of the mass-wheel system after the wheel has turned one revolution.



Solutions

1. Cross product of 5i + 6j and -3i + -2j

$(5\hat{\imath}+6\hat{\jmath})\times(-3\hat{\imath}\times-2\hat{\jmath})$

Express in matrix form:

$$\begin{bmatrix}\hat{\imath} & \hat{\jmath}\\ 5 & 6\\ -3 & -2\end{bmatrix}$$

Determinant is

$$\begin{vmatrix}A_x & A_y\\ B_x & B_y\end{vmatrix}\hat{k}=\begin{vmatrix}5 & 6\\ -3 & -2\end{vmatrix}\hat{k}$$

Expanding determinant:

$(5\cdot-2)-(6\cdot-3)=(-10)-(-18)=8\hat{k}$

2. The correct answer is **c**. Torque, the "turning effect" produced by a Force applied to a moment-arm is calculated to $\tau = r x F$, or t = $\tau = rFsin\theta$, where θ is the angle between the vectors r and F. Here, each combination of wrench length and Force produces a net torque of LF except for answer c:

$$\tau = \mathbf{r}\times\mathbf{F} = rF\sin\theta$$

$$\tau = LF\sin 120 = LF\sin 60 = L2F\tfrac{\sqrt{3}}{2} = LF\sqrt{3}$$

3. Frictionless wheel contraption

a) To determine the tension in the cord, one approach to solving this problem involves doing Newton's Second Law analyses on the mass and the wheel, and solving for the tension in those two equations.

<u>**For the hanging mass:**</u>

$$\sum F = ma$$

$$F_g - T = Ma$$
$$Mg - T = Ma$$

<u>**For the wheel:**</u>

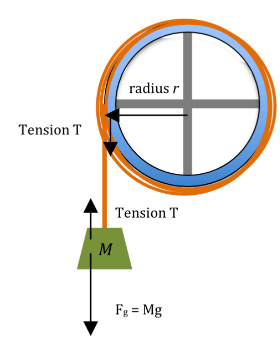
$$\sum \tau = I\alpha$$

$$r\times F = \left(\frac{5}{12}ML^2\right)\left(\frac{a}{r}\right)$$

$$rT = \left(\frac{5}{12}ML^2\right)\left(\frac{a}{r}\right)$$

$$r = \frac{L}{2}, so$$

$$a = \frac{3T}{5M}$$



Substitute and combine the two relationships to solve for T to get $T = \frac{5}{8}Mg$

b) Use previous relationships to get acceleration, and then angular acceleration.

$$a = \frac{3T}{5M} = \frac{3\left(\frac{5}{8}Mg\right)}{5M} = \frac{3}{8}g$$
$$\alpha = \frac{a}{r} = \frac{\frac{3}{8}g}{L/2} = \frac{3}{4}\frac{g}{L}$$

c) Because the system is accelerating constantly, we can use a kinematic relationship.

$$v_f^2 = v_i^2 + 2a\Delta x$$
$$v_f = \sqrt{0 + 2\left(\frac{3}{8}g\right)(2\pi r)}$$
$$r = L/2, \text{ so}$$
$$v_f = \sqrt{2\left(\frac{3}{8}g\right)\left(2\pi\left(\frac{L}{2}\right)\right)} = \sqrt{\frac{3}{4}\pi g L}$$

d) To get the angular momentum of the system, we'll need to include both the moving mass and the rotating wheel:

$$L_{system} = L_{mass} + L_{wheel}$$
$$L_{system} = r \times mv + I\omega$$
$$L_{system} = rMv + I\left(\frac{v}{r}\right)$$

Then using answers from previous solutions we can solve:

$$L_{system} = \left(\frac{L}{2}\right)M\sqrt{\frac{3}{4}\pi g L} + \left(\frac{5}{12}ML^2\right)\left(\frac{\sqrt{\frac{3}{4}\pi g L}}{L/2}\right)$$
$$L_{system} = \frac{4}{3}ML\sqrt{\frac{3}{4}\pi g L} = ML\sqrt{\frac{4}{3}\pi g L}$$